Damped-amplitude & polynomial frequency model for short-time ambient vibrations of a building

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Abstract

The modelling of signal in the context of multiple components, few samples, strong non-stationarity and non-linearity is a difficult problem. For signals in this category, classical methods such as Fourier-based techniques, time-frequency representation, and high order ambiguity function suffer from limitations of various kinds. For this concern, some other techniques which avoid these limitations should be applied. A model based on polynomial approximation of both the instantaneous amplitude and frequency had been proved to be a favorable choice. With consideration of adapting to ambient vibration signals where the amplitude is damped, we propose to use a new model of which the amplitude is approximated by damped exponentials. Meanwhile, the instantaneous frequency is represented by low-order orthonormal polynomials in order to track the strong local variations. Then the parameters estimation is carried out via a maximum likelihood procedure followed by a stochastic optimization method. For the purpose of faster convergence, the adaptive simulated annealing is employed which permits a more flexible temperature tuning. In order to study the behavior of the proposed algorithm, we then detail its performance based on simulated multi-component signals. Cramer-Rao bounds are recalculated and compared with those obtained by the model previously proposed, where the amplitude is based on polynomial approximation. Analysis of the frequency resolution between two closely spaced components is also shown as another important evidence of performance. In order to achieve a direct physical interpretation in real world applications, the proposed amplitude-frequency model is transformed to the physical damping model, under which the estimated signal is decomposed into time-varying resonance frequencies and damping ratios. The algorithm is further applied on ambient vibrations of buildings as a local analysis. Results are discussed in agreement with the model of a dissipative dynamic system, which corresponds to the ambient vibrations of a building.

1. Introduction

In earthquakes, the number of casualties is closely correlated to the structural damage of the building. The project URBASIS aims to evaluate the vulnerability of urban buildings by signal processing techniques. In this paper, the modal parameters, such as the resonance frequency and the damping ratio, are estimated from ambient vibration signals of buildings recorded by accelerometers simultaneously at longitudinal, transverse and vertical directions.

Figure 1 shows that the modal parameters of the ambient vibration signals present local variations. In this paper, the modal parameters are no longer assumed to be time-invariant values as for Frequency Domain Decomposition (FDD) and peak-picking methods based on power spectral density, instead, they are regarded as time-varying functions even in a few seconds. Thus, a method which permits the tracking of their local variations is required.

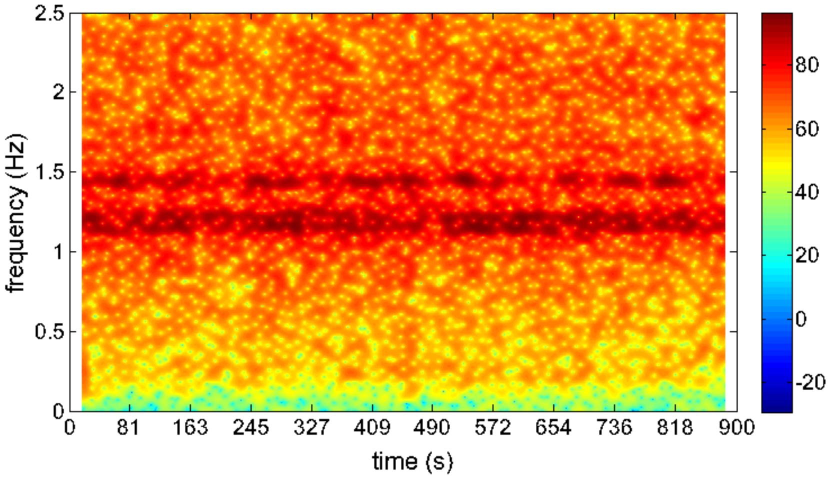


Figure 1. Example of ambient vibration signal: the vertical recording measured at the top of Grenoble City Hall. Spectrogram calculated with Hann window of 32 s.

In this paper, a damped-amplitude model is proposed in the context of multi-component ambient vibration signals, and further for seismic signals where the variation of the modal parameters is more intense. The signal is represented as a sum of components with time-varying frequency and exponentially damped amplitude. This model is of great interest thanks to a faster computation and a direct extraction of the damping factor. For the purpose of comparison, referred to as an indirect approach in this paper, the polynomial amplitude model we proposed in [4] will be applied as well, the parameters being transformed in order to finally model a damped amplitude.

Therefore, we address the issue of modeling short-time signals having both strong non-stationary frequency and damping amplitude. Classical techniques, which are simple and fast, have limitations in this context [2][3]. Instead, a maximum likelihood approach is applied for parameter estimation. The non-linearity of the likelihood function compels the use of stochastic optimization techniques such as simulated annealing, implemented by Monte-Carlo random sampling combined with a Metropolis acceptance rule. In [4][5][6], this optimization has been proved to be quite efficient as a solution for short-time polynomial models.

To speed up the estimation and to solve the non-linear optimization problem, we consider the Adaptive Simulated Annealing (ASA). Each multi-dimensional search must take into account the varying sensitivities of different parameters. At any given annealing-time, the adaptive simulated annealing described in [7] stretches out the range over which the relatively insensitive parameters are being searched, with respect to the ranges of the more sensitive parameters. In addition, this way of doing induces a gain in computing time.

In section 2, the model is defined and the constraints of the modulation functions are discussed. Section 3 sums up the parameter estimation and the new Cramer-Rao bounds. In section 4, the performance of the proposed algorithm is analyzed on simulated signals; the frequency resolution limit and the ability to separate components of differents amplitudes are also presented. Section 5 presents the application on real-world ambient vibration signals measured on the top of the Grenoble City Hall; the results are compared to classical methods used in seismic analysis. Finally, the conclusion is drawn in section 6.

2. Damped amplitude & polynomial frequency model

Let $y[n]$ be a discrete time process consisting of a deterministic multi-component process $s[n]$ embedded in an additive white Gaussian noise $e[n]$ with zero mean and variance σ^2,

$$\begin{aligned} & y[n] = s[n] + e[n] \quad \text{with} \quad s[n] = \sum_{i=1}^{K} A_i[n] e^{j\Phi_i[n]}, \\ & \Phi_i[n] = 2\pi \left(\sum_{k=-N/2}^{n} F_i[k] - \sum_{k=-N/2}^{0} F_i[k] \right) + \varphi_{0,i} \quad \text{and} \quad F_i[n] = \sum_{m=0}^{M_f} f_{m,i} \cdot g_m[n], \end{aligned} \tag{1}$$

where $-\frac{N}{2} \leq n \leq \frac{N}{2}$ with N even, K the number of components. $A_i[n]$ is the time-varying amplitude. As in [7], $\Phi_i[n]$ is the instantaneous phase of the i^{th} component centered in the middle of the observation window to minimize the error of estimation, thus $\varphi_{0,i} = \Phi_i[0]$. The instantaneous frequency is approximated by discrete orthonormal polynomial functions at maximum third order. At m^{th} order, $f_{m,i}$ and $g_m[n]$ are the frequency parameter and the orthonormal polynomial respectively.

In this paper, we intend to study a new model for the amplitude in correspondence with many real-world signals where $A_i[n] = \beta_i e^{-\alpha_i n}$. The initial amplitude β_i and the damping coefficient α_i characterize the amplitude of the i^{th} component. In order to proceed with estimation of β_i, α_i, φ_0 and of the $(M_f + 1)$ frequency parameters $f_{m,i}$, i.e. $(M_f + 4)$ parameters, the following constraints are imposed: $0 < F_i[n] < \frac{F_s}{2}$ with F_s the sampling frequency, $N + 1 > K \times (M_f + 4)$, and $\Phi_i[n]$ does not include any discontinuities. With regard to real-world data, β_i and α_i are constrained to be strictly positive.

An intrinsic error of the complex model with damped amplitude has to be mentioned.

The model defined in (1) does not always satisfy Bedrosian conditions because the exponential amplitude can present a wide-band spectrum. However, in actual problems that we investigate, frequencies are at about 1 Hz, and damping coefficients α_i are usually trivial ($<10^{-1}$) [9], then the -3 dB spectral bandwidth of the damped amplitude ($8.08\,\alpha_i$ Hz) is very narrow. This error is thus negligible even for the low frequency signal that we process.

3. Parameter estimation and Cramer-Rao bounds

For the signals that this paper deals with, classical methods as in [10] are hardly applicable in the short time context. More sophisticated methods should be used in order that parameters can be correctly estimated with very few samples. We proposed to use the maximum likelihood approach and the adaptive simulated annealing to solve the non-linear optimization problem which is impossible to be done by classical methods. The Cramer-Rao bounds of the modulation functions under the proposed model are also discussed.

3.1 Parameter estimation algorithm

In this section, we discuss the approximation of a model by low order polynomials which intend to track locally highly non-stationary modulations. The signals are of short time duration, the sample number N ranges approximately from 30 to 100. Let us consider the instantaneous frequency $F_i\left[n\right]$ (1) to be approximated by an orthonormal polynomial at M_f^{th} order. Then the parameters of each component in (1) form a vector

$$\boldsymbol{\theta}_i=\left[\boldsymbol{\theta}_{A_i},\varphi_{0,i},\boldsymbol{\theta}_{F_i}\right]=\left[\beta_i,\alpha_i,\varphi_{0,i},f_{0,i},\ldots,f_{M_f,i}\right], \tag{2}$$

where $1\le i\le K$, so that the parameters of all the components are

$$\boldsymbol{\theta}=\left[\theta_{i,j}\right]_{K\times(M_f+4)}=\left[\boldsymbol{\theta}_1^{\mathbf{T}},\ldots,\boldsymbol{\theta}_K^{\mathbf{T}}\right]. \tag{3}$$

Each element $\theta_{i,j}$ in $\hat{\boldsymbol{\theta}}$ corresponds to the j^{th} parameter of the i^{th} component. We consider a maximum likelihood estimation of $\boldsymbol{\theta}$, which corresponds to a least square approach under the hypothesis of a white Gaussian noise,

$$\hat{\boldsymbol{\theta}}=\underset{\boldsymbol{\theta}\in\mathbb{R}^{K\times(M_f+4)}}{\arg\min}\sum_{n=-N/2}^{N/2}\left|y\left[n\right]-s\left[n\right]\right|^2 . \tag{4}$$

In [6], optimal and sub-optimal algorithms were developed to balance between the precision and the computation cost. The sub-optimal approach can be affected when one component has distinctly weaker amplitude than the others. So thereafter, all examples are calculated by the optimal approach. We assume that the number of components K and the polynomial order of each component M_f are *a priori* known and remain unchanged. This assumption is justified by the short-time duration.

In this paper, an enhanced version of the simulated annealing, referred to as adaptive simulated annealing [7], is applied to solve (4). For this meta-heuristic method, the steps of the parameter initialization and of the estimation procedure is close to simulated annealing [4]. Compared to previous publications, the enhancements are not only in taking into account the damped amplitude model, but also in introducing some extra data-driven configurations. The method is "adaptive" since the temperatures which control the generation and acceptation of the candidates are dependent of the data during the convergence process. This contributes to a faster convergence to the global optima and a simpler parameter tuning.

Moreover, a step named "re-annealing" is included in adaptive simulated annealing, in order that an uniform search range of all parameters could be attained by regulating the parameter generation temperature. Important temperature-control configurations are N_{gen}, N_{accept} and c, which control the temperature of parameter generation, the temperature of candidate parameter acceptance and the speed of temperature decrease respectively. The details of the method can be found in [13].

3.2 Cramer-Rao Lower Bounds

We propose to calculate the Cramer-rao bounds for the model defined in (1,2,3) under the discrete orthonormal polynomial base we used as in [6], not only for the parameters but also for the modulation functions. The Fisher information matrix of the i^{th} component defined by (1) is given as

$$\boldsymbol{I}_{\boldsymbol{\theta}_{A_i,F_i}} = \frac{1}{\sigma^2}\Re\left\{\begin{pmatrix} \boldsymbol{I}_{\boldsymbol{\theta}_{A_i}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_{\boldsymbol{\theta}_{F_i}} \end{pmatrix}\right\} \text{ with } \boldsymbol{I}_{\boldsymbol{\theta}_{A_i}} = \begin{pmatrix} \sum_{n=-N/2}^{N/2} 2e^{-2\alpha_i n} & -\sum_{n=-N/2}^{N/2} 2n\beta_i e^{-2\alpha_i n} \\ -\sum_{n=-N/2}^{N/2} 2n\beta_i e^{-2\alpha_i n} & -\sum_{n=-N/2}^{N/2} 2n^2\beta_i^2 e^{-2\alpha_i n} \end{pmatrix}. \tag{5}$$

Denote $\eta_m[n] = \sum_{k=-N/2}^{n} g_m[k]$ as the numerical integration of the orthonormal polynomial, $\boldsymbol{I}_{\boldsymbol{\theta}_{F_i}}$ is a block matrix of $\left(M_f+1\right)\times\left(M_f+1\right)$ dimension, with elements

$$\boldsymbol{I}_{h,l} = \sum_{n=-N/2}^{N/2} 2\eta_h[n]\eta_l[n]\beta_i^2 e^{-2\alpha_i n}; 1 \le h,l \le M_f + 4. \tag{6}$$

Then Cramer-Rao bounds of the amplitude $CRB_{A_i}[n]$ and of the frequency $CRB_{F_i}[n]$ in the non-biased case are

$$CRB_{A_i}[n] = \frac{\sigma^2}{2}\boldsymbol{d}_i^{\dagger}\left\{\boldsymbol{I}_{\theta_{A_i}}^{\dagger}\boldsymbol{I}_{\theta_{A_i}}\right\}\boldsymbol{d}_i \text{ and } CRB_{F_i}[n] = \frac{\sigma^2}{2}\boldsymbol{h}_i^{\dagger}\left\{\boldsymbol{I}_{\theta_{F_i}}^{\dagger}\boldsymbol{I}_{\theta_{F_i}}\right\}\boldsymbol{h}_i\ , \tag{7}$$

with $\boldsymbol{d}_i = \left[e^{-\alpha_i n}, \beta_i n e^{-\alpha_i n}\right]^{\dagger}$, $\boldsymbol{h}_i = \left[g_0[n],\ldots,g_{M_f}[n]\right]^{\dagger}$, and $[\cdot]^{\dagger}$ the conjugated transpose.

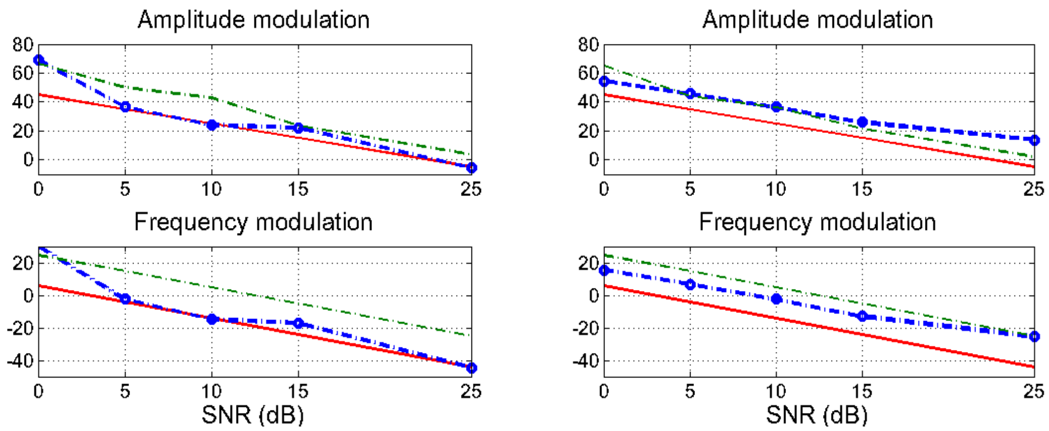
In the next section, these bounds will be used to compare the performance of the proposed algorithm to what we defined the indirect approach.

4. Analysis based on simulated signals

In this section, the performance of the proposed algorithm is compared to the indirect one in the sense of Cramer-Rao bounds (7) via Monte-Carlo simulations. Given the property of signals dealt in this paper, signals of which the component frequencies can be very close, and at the same time, a component can be relatively much weaker than the others, the frequency resolution limit and the weak amplitude tolerance limit are developed as a reference of adaptability for real-world applications.

4.1 Performance of the algorithm compared to the indirect one

The Cramer-Rao Bounds (7) of the proposed method are compared the results got with a 2-component signal, with quadratic frequency modulation and damped amplitude. In accordance with real-world data, the damping ratios are taken in the order of several percent. Figure 2 shows Cramer-Rao bounds of both components with a SNR varying from 0 to 25 dB, averaged among 100 noise runs. We observe that these bounds of the algorithm proposed are lower than those obtained by the indirect algorithm which induced an extra error caused by amplitude regression.



(a) First component (b) Second component

Figure 2. MSEs and CRBs in dB for a 2-component simulated signal, 33 points sampled at 1Hz. MSEs (-.o blue). CRBs of the algorithm proposed (– red) and of the indirect algorithm (- - green).

4.2 Frequency resolution limit

In real-world applications, it is usual that signals are composed of components which are closely spaced in frequency. Therefore, the frequency resolution limit of the proposed algorithm is explored in this section using a signal simulated by 9-parameter sets. Each signal consists of two components with constant amplitude and quadratic frequency modulation. For each component, the parameter vectors as defined in (2) are $\boldsymbol{\theta}_1 = \left[10, 0, 0.4, f_{1,0}, -0.2, -0.5\right], \boldsymbol{\theta}_2 = \left[10, 0, 0.8, f_{2,0}, -0.2, -0.5\right]$.

In order to approach gradually to the frequency resolution limit, $f_{1,0}$ is fixed at 2, while $f_{2,0}$ is variable from 1.55 to 1.95 in step of 0.05. The simulation is carried out with a SNR of 15 dB, N equal to 32 and a sampling frequency of 1 Hz.

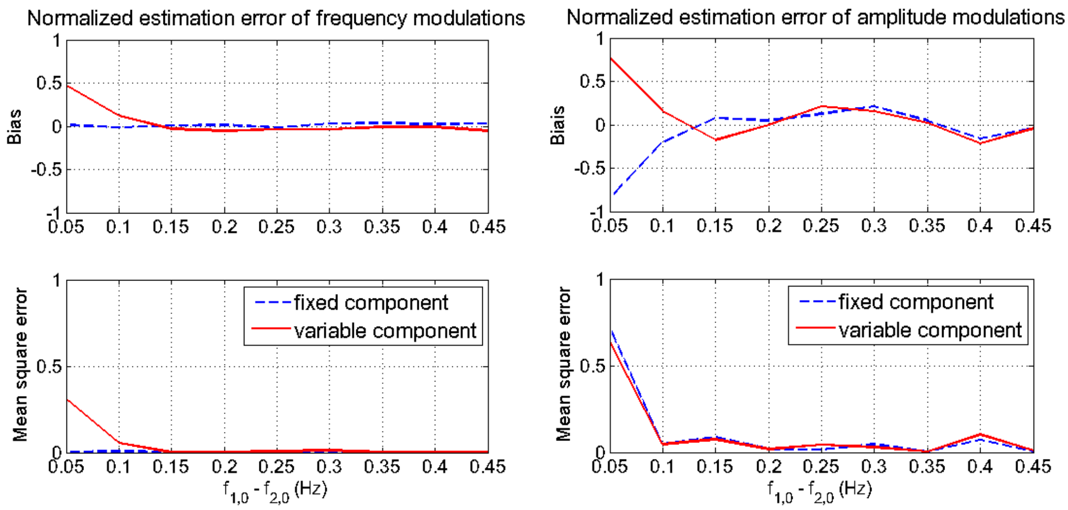


Figure 3. Normalized bias (up) and mean square error (down) of frequency resolution test

In figure 3, the normalized bias and the MSE of the amplitude and frequency modulations are shown as an average of 20 noise runs. Satisfying results can be obtained for $f_{2,0} - f_{1,0} \geq 0.15$ where the frequency distance is larger than 0.0261 Hz. In the extreme case, where $f_{2,0} - f_{1,0} \leq 0.1$, the two simulated components are too close to be separated, therefore, one component is estimated in the middle of the two components with twice amplitude, while another is visible in lower frequency with very weak amplitude.

In this paper, the minimum frequency distance of the signals presented in section 6 is about 0.06Hz ($f_{2,0} - f_{1,0} = 0.47$), for which the frequency resolution of the proposed algorithm is sufficient according to figure 3. Furthemore, in this application, the resolution of a Fourier transform is 0.12 Hz and that of a spectrogram is only 0.225 Hz with a Hann window used in both cases.

4.3 Weak amplitude tolerance limit

In this paper, the signals of interest involve multiple components of which the amplitudes are drastically different. In this section, the weak amplitude tolerance limit is investigated using a two-component signal with constant amplitude and quadratic frequency modulation. For each component, the parameter vectors as defined in (2) are $\boldsymbol{\theta}_1 = [\beta_1, 0.03, 0.4, 2, 0, 0]$, $\boldsymbol{\theta}_2 = [\beta_2, 0.03, 0.8, 1, 0, 0]$, β_1 is fixed at 10, β_2 decreases from 9 to 1 in step of 1. The simulation is carried out with a SNR of 15 dB, N equal to 32 and a sampling frequency of 1 Hz. Figure 4 shows the results averaged among 20 noise runs. The distortion starts to clearly increase at an amplitude ratio β_1/β_2 of 5, which corresponds to a signal power ratio of 25. This value can be considered as a limit in the same conditions.

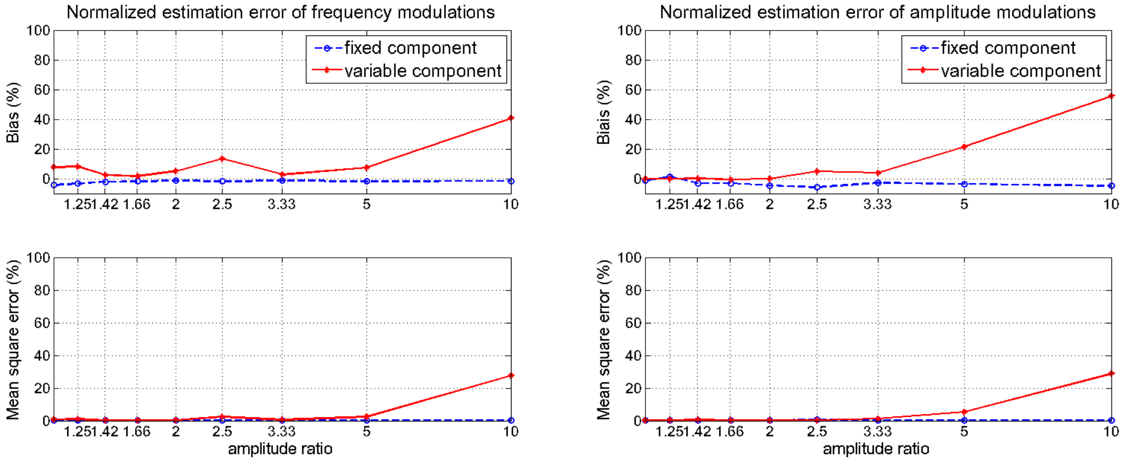


Figure 4. Normalized bias and mean square error of weak amplitude tolerance test. The "amplitude ratio" is calculated as β_1/β_2 .

5. Application on ambient vibration signals

Civil architectures are permanently excited by some natural solicitation sources, for example, the background seismic noise from the earth, the wind and sea wave, and the internal sources (human steps, vehicles, rotating machines). These different excitations induce various types of vibrations in the building, which are referred to as ambient vibrations. The ambient vibration signal is the impulse response of a dissipative system under quasi-stationary excitations. The signal analysed in this section is recorded using multiple sensors placed in one or several stories to measure simultaneously the vibrations in three directions, longitudinal, transverse and vertical ones. The signals studied in this section are measured at the top of Grenoble City Hall in France [9][11].

Figure 5. Grenoble city hall, France.

A physical model adapted to such signals can be written as [9][11]

$$s[n]=\sum_{i=1}^{K}\beta_i\cdot\exp\left(j\varphi_{0,i}+\sum_{k=-N/2}^{n}\lambda_i[k]\right),\ \lambda_i[k]=2\pi\left(-\xi_i[k]\upsilon_i[k]+j\upsilon_i[k]\sqrt{1-\xi_i^2[k]}\right).\quad(8)$$

Under such model, the evaluation of the structural variation of the building focuses on two modal parameters: the time-varying damping ratio $\xi_i[k]$ and the time-varying resonance frequency $\upsilon_i[k]$. Methods usually used by seismologists are based on the assumption that the vibration signals are quasi-stationary, thus the temporal variation of the modal parameters are neglected. In a non-stationary approach as in [11], the proposed method permits the analysis of time varying modal parameters, where $\xi_i[k]$ and $\upsilon_i[k]$ are functions of time.

5.1 Seismic signal processing by Frequency Domain Decomposition

Brincker et al. [12] described a frequency domain technique known as the Frequency Domain Decomposition (FDD) for the modal identification of output-only systems. The modal parameters are estimated as time-invariant values using all the available measurements. Belonging to the category of non-parametric method, the principle is to decompose the cross power spectral density matrix of the measurements into singular values which are further related to independent degrees of freedom. The first singular value is regarded as the power spectral density of all Single Degree Of Freedom (SDOF) responses, whose peaks locate the resonance frequencies [9]. Figure 6 shows the FDD results of the ambient vibration signals recorded on the top of Grenoble City Hall for a sampling frequency of 5 Hz.

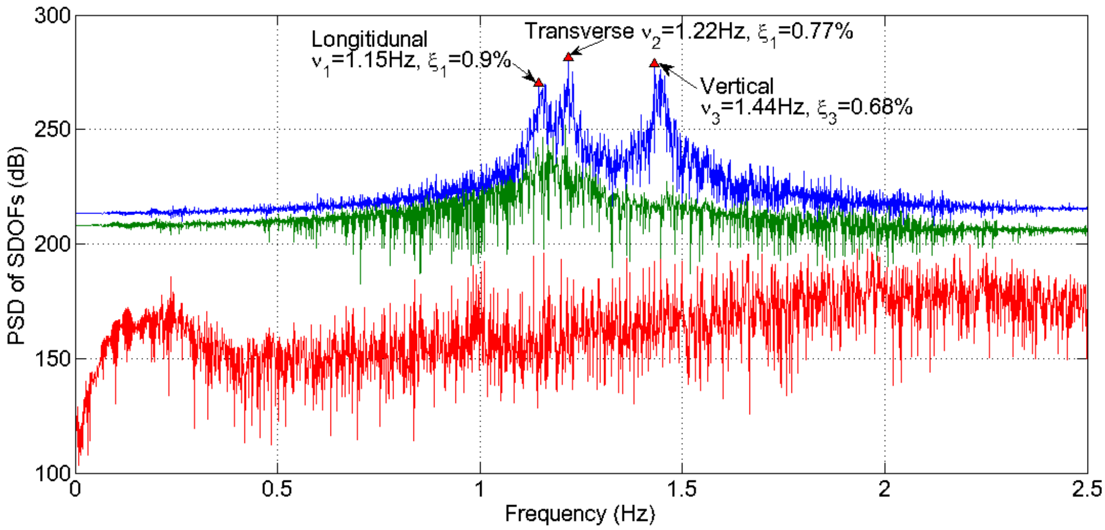


Figure 6. Singular values (fisrt, second, third) and the modal parameters estimated of FDD approach for ambient vibrations of Grenoble City hall

In figure 6 is calculated using all the three directions of measurement, the estimated resonance frequency coincide well with prior analysis [9]. In order to calculate the corresponding damping ratios, the “spectral bells” around the resonance frequency peaks in the first singular value are extracted using the Modal Assurance Criterion [9], and then taken back to the time domain by the inverse Fourier transform to generate the finite impulse response of the mode, namely the autocorrelation function of SDOFs. The estimation of the damping ratio of the transverse mode is illustrated in figure 7.

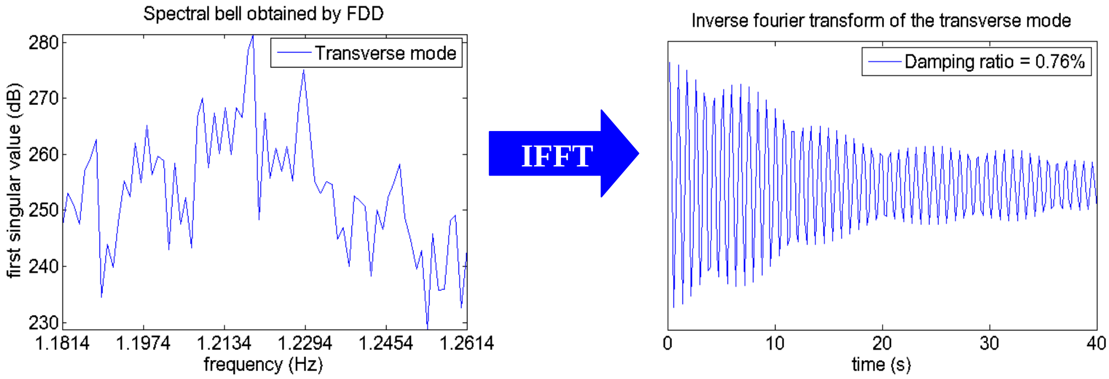


Figure 7. Left: Spectral bell belonging to the transverse mode in figure 6.
Right: Inverse Fast Fourier transform (IFFT) of the bell.

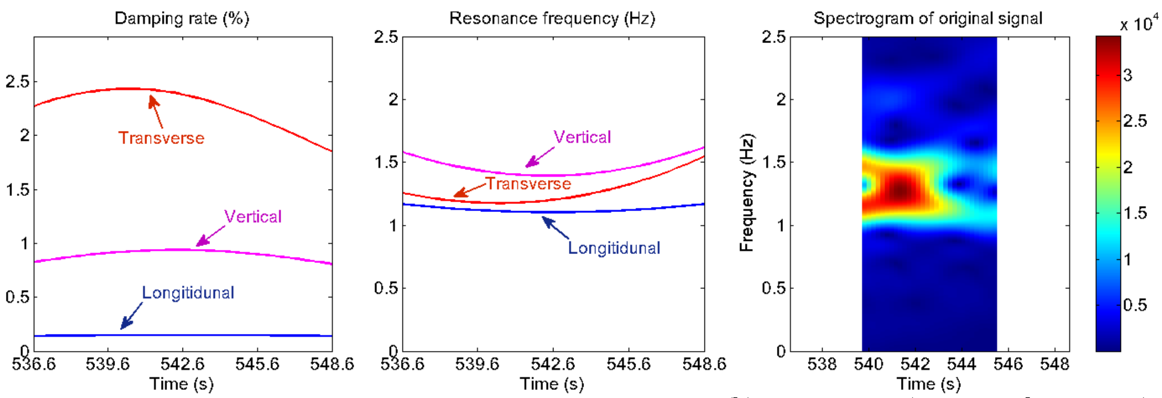
Each damping ratio is finally calculated from a regression of the envelop of such response in logarithmic scale with respect to time. The conclusion drawn by FDD had been proved to be a realiable global estimation under a stationary assumption [9].

5.2 Analysis using the proposed algorithm

One preliminary assumption taken by FDD is the stationarity of the signal. However, the correctness of such method can no longer be justified in non-stationary and short time cases. Being an advantage of great interest, the proposed method permits to track the local variation of these modal parameters within several seconds in each direction. Combined with the conclusions of sections 4.2 and 4.3, the proposed algorithm is qualified to attain a good estimation. According to (8), the parameters of each component in (1) are directly connected with the modal parameters of a mode, as

$$\upsilon_i[k] = \frac{1}{2\pi}\sqrt{\left(2\pi F_i[k]\right)^2 - \alpha_i^2}\ ,\ \xi_i[k] = \alpha_i / 2\pi\upsilon_i[k]. \tag{9}$$

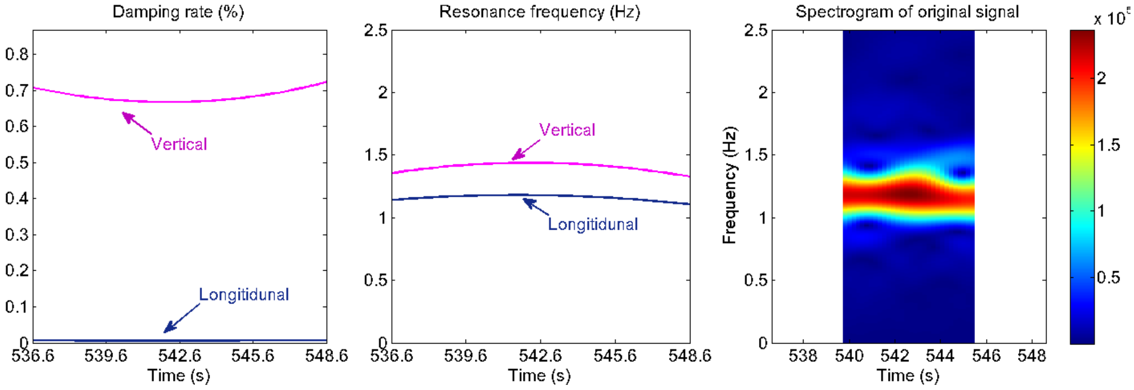
At each direction the signal from 536.6 s to 648.6 s is modelled using the proposed method with N_{gen}*=80*, N_{accept}*=60*, *c=3*. Figures 8 to 10 present the estimated modal parameters and the corresponding spectrogram of which frequency resolution is 0.225 Hz and time resolution is 6.4 s, that in the three directions.



(a) Estimation by the proposed algorithm (b) Spectrogram (Hann window, 32 pts)

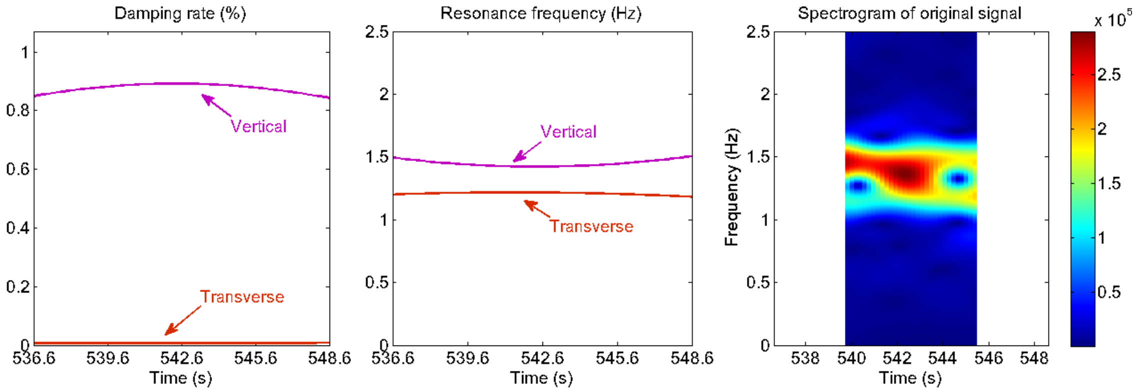
Figure 8. Results calculated on [536.6 s, 548.6 s] of the vertical recording.

Normally, the most accurately estimated mode is the one recorded in its own direction. In figure 8, the vertical and the longitudinal modes are correctly identified whereas the estimation of the transverse mode is disturbed. In the vertical measurement, all the modes are visible in the same time. In other directions, only 2 modes are visible in this time segment.



(a) Estimation by the proposed algorithm (b) Spectrogram (Hann window, 32 pts)

Figure 9. Results calculated on [536.6 s, 548.6 s] of the longitudinal recording.



(a) Estimation by the proposed algorithm (b) Spectrogram (Hann window, 32 pts)

Figure 10. Results calculated on [536.6 s, 548.6 s] of the transverse recording.

Figures 8 to 10 show results of the algorithm proposed over the time duration of 12s. The resonance frequencies coincide with the average values obtained by FDD and the modal parameters are varying throughout this time duration. The damping ratios estimated show differences compared with the average values estimated by FDD in figure 6. This is due to the existence of non-stationarity even in 12 s.

6. Conclusion

In this paper, the application of the damped-amplitude and polynomial-frequency model is studied in the context of ambient vibrations. This model is applied on a short-time segment of the entire signal. The parameter estimation is based on the maximization of likelihood function optimized by adaptive simulated annealing. By calculating and analyzing the Cramer-Rao bounds, it is shown that the estimation of both the amplitude and the frequency modulation functions are improved compared to a polynomial-amplitude model. The frequency resolution limit is calculated from 2-component simulated signals. The proposed method is capable to track the variation of multi-component signals and directly identify the modal parameters for each component. By that way, the ambient vibrations of a building and more particularly their damping coefficients have been characterized over a very short time of 12 s (60 samples), which has never been done before.

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